both methods simultaneously whenever there is any doubt in practice.

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Crystallography, Geometry and Physics in Higher Dimensions. VIII. The WPV Symbols of the 38 Cyclic Crystallographic Point Symmetry Groups in the Five-Dimensional Space $\mathbb{E}^5$

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Abstract

This paper is the first of a series of three devoted to crystallography in the five-dimensional space $\mathbb{E}^5$. The 38 types of point symmetry operations (PSO for short) are described *i.e.* 19 types of PSO$^+$s or rotations and 19 types of PSO$^-$s or improper rotations; each of them generates a cyclic point group. A WPV (Weigel, Phan, Veysseyre) symbol is given both to the PSOs and to the cyclic groups. There is a generalization of the well known symbols of $\mathbb{E}^3$. For instance, $\bar{6}$ is the symbol of a point group of $\mathbb{E}^3$ (and $\mathbb{E}^4$), and $\bar{6}$ has application in $\mathbb{E}^5$ (and $\mathbb{E}^6$); but new symbols such as $\bar{\bar{6}}$, $\overline{\overline{66}}$ are also required.

Introduction

Before giving the geometrical name of the 23 crystal families of the space $\mathbb{E}^5$ and the WPV symbol* of their holohedries, *i.e.* the crystallographic point group of their empty lattice, we must list all the types of crystallographic point symmetry operations (cr PSOs for short). Indeed these cr PSOs generate the cr point groups and determine their WPV symbols.

To start we recall the number of types of cr PSOs and their WPV symbols in spaces of dimension less than five (Veysseyre & Weigel, 1989)

2 cr PSOs in $\mathbb{E}^1$: 1 and m;
6 cr PSOs in $\mathbb{E}^2$: 1, 2, 3, 4, 6 and m;
10 cr PSOs in $\mathbb{E}^3$: 1, 2, 3, 4, 6 and m, $\bar{1}, \bar{3}, \bar{4}, \bar{6}$;
24 cr PSOs in $\mathbb{E}^4$: 1, 2, 3, 4, 6, $\bar{1}_4$, 24, 26, 32, 33, 43, 44, 46, 63, 66, 55, 1010, 88, 1212 and m, $\bar{1}, \bar{3}, \bar{4}, \bar{6}$.

Let us remember that 3^1_{xy}, 3 for short, is the elementary rotation through the angle $2\pi/3$ in the plane xy about a point in $\mathbb{E}^2$, about the z axis in $\mathbb{E}^3$ and about the 3-dimensional space ztu in $\mathbb{E}^5$.

On the other hand, the double rotation $8^1_{xy}8^3_{zt}$, 88 for short, about a point in $\mathbb{E}^4$ (and about the axis u in $\mathbb{E}^5$) is the commutative product of two rotations through the angle $2\pi/8$ in the plane xy about the plane zt, and through the angle $6\pi/8$ in the plane zt about the plane xy. Let us recall that the two planes xy and zt are orthogonal and that they intersect at only one point.

I. Crystallographic point symmetry operations of $\mathbb{E}^5$

The number of types of cr PSOs is well known in $\mathbb{E}^5$ (Hermann, 1949; Weigel, Veysseyre, Phan, Effantin & Billiet, 1984). Indeed there are 19 types of cr PSO$^+$s and 19 types of cr PSO$^-$s as in a space of odd dimension the number of cr PSO$^+$s and of cr PSO$^-$s are equal. The PSO$^+$s are the proper rotations, and the PSO$^-$s are the improper rotations. For example 3 is the threefold rotation and hence it is a PSO$^+$; $\bar{3}$ is the threefold rotation-inversion or a rotation-reflection through the angle $-2\pi/6$, it is a PSO$^-$. A homothetie of ratio (-1) can be a PSO$^+$ if its dimension is an even number, such as $\bar{1}_4$ for instance, or a PSO$^-$ if its dimension is an odd number, *e.g.* $\bar{1}_3, \bar{1}_5 \ldots$

* WPV: Weigel, Phan & Veysseyre (1987) generalized Hermann-Mauguin symbols.

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If we return to the space $\mathbb{E}^4$, we know that there are 19 types of PSO$^+$s and 5 types of PSO$^-$s.

Our purpose is to assign a WPV symbol to the new types of PSOs of $\mathbb{E}^5$. As a matter of fact, we represent the space $\mathbb{E}^5$ as a direct sum of two orthogonal subspaces:

$$\mathbb{E}^5 = \mathbb{E}^4 \oplus \mathbb{E}^1.$$

Then the matrix of a PSO is matrix 1 with respect to a correct basis, where A is the matrix associated with a PSO$^+$ of the subspace $\mathbb{E}^4$ and ε equals either 1 for a PSO$^+$ of $\mathbb{E}^5$ or -1 for a PSO$^-$ of $\mathbb{E}^5$.

$$\left(\begin{array}{cccc|c} & & & & 0 \\ & A & & & 0 \\ & & & & 0 \\ & & & & 0 \\ \hline 0 & 0 & 0 & 0 & \varepsilon \end{array}\right)$$

Matrix number 1. General PSO of $\mathbb{E}^5$.

(1) *The* 19 *types of PSO$^+$s of* $\mathbb{E}^5$

As they correspond to ε equal to 1 in the previous decomposition, they are all the PSO$^+$s of $\mathbb{E}^4$ and consequently they have the same WPV symbols. We can illustrate this with the example of the double rotation denoted $4^1_{xy}4^1_{zt}$ and described by matrix 2. Of course, all these 19 PSO$^+$s are polar PSO$^+$s of $\mathbb{E}^5$ (Veysseyre & Weigel, 1989).

$$\left(\begin{array}{cccc|c} 0 & \bar{1} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{1} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

Matrix number 2. Double rotation $4^1_{xy}4^1_{zt}$.

(2) *The* 19 *types of PSO$^-$s of* $\mathbb{E}^5$

These correspond to ε equal to -1 in the previous decomposition. We can classify these PSO$^-$s in the following way:

(a) A is the unit matrix of $\mathbb{E}^4$. The corresponding matrix in $\mathbb{E}^5$ is the diagonal matrix 3 where the unwritten terms are zeros. This PSO has the symbol m_u and it is a reflection through the mirror-hyperplane ($xyzt$). It is the *first type of PSO$^-$* of $\mathbb{E}^5$.

$$\left(\begin{array}{cccc|c} 1 & & & & 0 \\ & 1 & & & 0 \\ & & 1 & & 0 \\ & & & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & \bar{1} \end{array}\right)$$

Matrix number 3. Reflection m_u.

(b) A is the matrix associated with the PSO $\bar{1}_4$. Obviously the corresponding matrix of $\mathbb{E}^5$ is the reverse of the matrix identity and this PSO$^-$ has the symbol $\bar{1}_5$. In the space $\mathbb{E}^3$ we denoted the PSO$^-$ $\bar{1}_3$, or $\bar{1}$ for short; in the same way the PSO$^-$ $\bar{1}_5$ of $\mathbb{E}^5$ will be denoted $\bar{\bar{1}}$ (matrix 4).

$$\left(\begin{array}{cccc|c} \bar{1} & & & & 0 \\ & \bar{1} & & & 0 \\ & & \bar{1} & & 0 \\ & & & \bar{1} & 0 \\ \hline 0 & 0 & 0 & 0 & \bar{1} \end{array}\right)$$

Matrix number 4. Total homothetie $\bar{\bar{1}}$.

(c) A is the matrix of an elementary rotation 4^1_{xy} for instance (matrix 5). It is also the same type of PSO$^-$ of $\mathbb{E}^4$ as the previous one and its WPV symbol is $4^1_{xy}m_u$. In this way we find *four types of PSO$^-$* of $\mathbb{E}^5$ which have the following symbols:

$$2_{xy}m_u = \overline{1}_{xyu}$$
$$3^{\pm 1}_{xy}m_u = \overline{6^{\mp 1}_{xy}}$$
$$4^{\pm 1}_{xy}m_u = \overline{4^{\mp 1}_{xy}}$$
$$6^{\pm 1}_{xy}m_u = \overline{3^{\mp 1}_{xy}}.$$

The first one is denoted $\bar{1}_{xyu}$ because it is a partial homothetie of ratio (-1) and of dimension 3; it is also an inversion through the plane zt.

$$\left(\begin{array}{cccc|c} 0 & \bar{1} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & \bar{1} \end{array}\right)$$

Matrix number 5. Rotation-reflection $4^1_{xy}m_u$.

(d) A is the matrix associated with a double rotation either through the same angle (matrix 6) or through two different angles (matrix 7). For these PSO$^-$s several notations are possible and correct.

$$\left(\begin{array}{cccc|c} 0 & \bar{1} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{1} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \bar{1} \end{array}\right)$$

Matrix number 6. Double rotation-reflection $\overline{\overline{\overline{4^{-1}_{xy}4^{-1}_{zt}}}}$.

$$\left(\begin{array}{cccc|c} \bar{1} & 0 & 0 & 0 & 0 \\ 0 & \bar{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{1} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \bar{1} \end{array}\right)$$

Matrix number 7. Rotation-reflection $\overline{\overline{\overline{4^{-1}_{zt}}}}$.

Table 1. *The* 19 *types of PSO⁻s of* $\mathbb{E}^5$

This table lists all the PSO⁻s of the five-dimensional space with their WPV symbols either with the reflection m or with the homothetie $\bar{1}$ or $\bar{\bar{1}}$ if possible.

m_u	
$\bar{1}_5$	$\bar{\bar{1}}$
$2_{xy}m_u$	$\bar{1}_{xyu}$
$3^1_{xy}m_u$	$\overline{6^{-1}_{xy}}$
$4^1_{xy}m_u$	$\overline{4^{-1}_{xy}}$
$6^1_{xy}m_u$	$\overline{3^{-1}_{xy}}$
$2_{xy}4^1_{zt}m_u$	$4^{-1}_{zt}\bar{\bar{1}}$
$2_{xy}6^1_{zt}m_u$	$3^{-1}_{zt}\bar{\bar{1}}$
$3^1_{xy}2_{zt}m_u$	$6^{-1}_{xy}\bar{\bar{1}}$
$3^1_{xy}3^1_{zt}m_u$	$6^{-1}_{xy}6^{-1}_{zt}\bar{\bar{1}}$
$4^1_{xy}3^1_{zt}m_u$	$4^{-1}_{xy}6^{-1}_{zt}\bar{\bar{1}}$
$4^1_{xy}4^1_{zt}m_u$	$4^{-1}_{xy}4^{-1}_{zt}\bar{\bar{1}}$
$4^1_{xy}6^1_{zt}m_u$	$4^{-1}_{xy}3^{-1}_{zt}\bar{\bar{1}}$
$6^1_{xy}3^1_{zt}m_u$	$3^{-1}_{xy}6^{-1}_{zt}\bar{\bar{1}}$
$6^1_{xy}6^1_{zt}m_u$	$3^{-1}_{xy}3^{-1}_{zt}\bar{\bar{1}}$
$5^1_{AB}5^{-2}_{CD}m_u$	$10^{-3}_{AB}10^1_{CD}\bar{\bar{1}}$
$8^1_{AB}8^3_{CD}m_u$	$8^{-3}_{AB}8^{-1}_{CD}\bar{\bar{1}}$
$10^3_{AB}10^{-1}_{CD}m_u$	$5^{-1}_{AB}5^2_{CD}\bar{\bar{1}}$
$12^1_{AB}12^5_{CD}m_u$	$12^{-5}_{AB}12^{-1}_{CD}\bar{\bar{1}}$

For instance the PSO⁻ given by matrix 6 can be written

$$4^1_{xy}4^1_{zt}m_u \quad \text{or} \quad \bar{1}_5 4^{-1}_{xy}4^{-1}_{zt} = \bar{\bar{1}}4^{-1}_{xy}4^{-1}_{zt}$$

or again

$$\overline{\overline{4^{-1}_{xy}4^{-1}_{zt}}}.$$

In the same way, the PSO⁻ given by matrix 7 can be written

$$2_{xy}4^1_{zt}m_u = \bar{1}_{xyu}4^1_{zt} = \bar{\bar{1}}4^{-1}_{zt} = \overline{\overline{4^{-1}_{zt}}}.$$

As there exist 13 types of double rotations in $\mathbb{E}^4$ which are 23; 24; 26; 33; 34; 36; 44; 46; 66; 55; 88; 1010; 1212, we find 13 *types of double rotation-inversions* in $\mathbb{E}^5$ (and in $\mathbb{E}^6$).

The 19 types of PSO⁻s of $\mathbb{E}^5$ are listed in Table 1.

(3) *Remarks*

(*a*) The total homothetie of ratio (-1) of dimension 4 is perfectly defined in the space $\mathbb{E}^4$ by the symbol $\bar{1}_4$ but in the space $\mathbb{E}^5$ we cannot write $\bar{1}_4$ or $\bar{1}$ without specifying the supports of these PSOs. We must write $\bar{1}_{xytu}$ for instance; then there is no ambiguity about the PSO.

More generally when we describe the total homothetie of ratio (-1) in the space $\mathbb{E}^n$ it is without interest to specify the support of this PSO and the symbol $\bar{1}_n$ is correct, but the support becomes necessary for a partial homothetie of dimension p in a space of dimension n when p is strictly smaller than n.

(*b*) For all the double rotation-reflections, two or three symbols are equally good as these PSO⁻s are also double rotation-inversions:

$$6^1_{xy}4^1_{zt}m_u = \bar{\bar{1}}3^{-1}_{xy}4^{-1}_{zt} = \overline{\overline{3^{-1}_{xy}4^{-1}_{zt}}}.$$

We remark that the notation with a double line above the symbol of the PSO can be used for a reflection-rotation. Indeed,

$$3^1_{xy}2_{zt}m_u = 3^1_{xy}\bar{1}_{ztu} = 6^{-1}_{xy}\bar{\bar{1}} = \overline{\overline{6^{-1}_{xy}}}.$$

As long as we consider the PSOs, the complete notation such as $6^1_{xy}4^1_{zt}m_u$ is the best one because it quickly gives all the supports of the elementary PSOs. But if we consider the cyclic group generated by these PSOs the reduced notation is best, as we shall explain in the next section.

II. Notation of the crystallographic cyclic point symmetry groups in $\mathbb{E}^5$

It is easy to prove that each type of cr PSOs of the space $\mathbb{E}^n$ generates a cyclic cr PSG of order p, p being the order of the considerated PSO; *i.e.* we can state the following general result:

In the n-dimensional space $\mathbb{E}^n$ *the number of cyclic crystallographic point groups is the same as the number of types of crystallographic point symmetry operations.*

In this way there are:

- 2 cyclic cr PSGs in $\mathbb{E}^1$;
- 6 cyclic cr PSGs in $\mathbb{E}^2$ among the 10 cr PSGs;
- 10 cyclic cr PSGs in $\mathbb{E}^3$ among the 32 cr PSGs;
- 24 cyclic cr PSGs in $\mathbb{E}^4$ among the 227 cr PSGs;
- 38 cyclic cr PSGs in $\mathbb{E}^5$ among the unknown number of cr PSGs.

We give an example to explain the construction of such a group. Let us consider the PSO denoted $4^1_{xy}4^1_{zt}m_u$; it generates the cyclic group with elements

$$4^1_{xy}4^1_{zt}m_u; \quad \bar{1}_{xyzt}; \quad 4^{-1}_{xy}4^{-1}_{zt}m_u; \quad 1.$$

It is of order 4 as the generator element.

Obviously, if we select another orientation of this space, the PSO will be denoted $4^{-1}_{xy}4^1_{zt}m_u$ for instance and the elements of the cyclic group will be

$$4^{-1}_{xy}4^1_{zt}m_u; \quad \bar{1}_{xyzt}; \quad 4^1_{xy}4^{-1}_{zt}m_u; \quad 1.$$

These two cyclic groups are isomorphic, and have the WPV symbol $\overline{\overline{44}}$. We have already explained the reduced notation of the PSO $4^1_{xy}4^1_{zt}m_u$.

Among the 38 PSOs of $\mathbb{E}^5$, 24 are PSOs of $\mathbb{E}^4$ and constitute the polar cr PSOs of $\mathbb{E}^5$ (Veysseyre & Weigel, 1989). They generate 24 cyclic polar PSGs of $\mathbb{E}^5$, and have symbols previously defined (Weigel, Phan & Veysseyre, 1987). They are listed in the first part of Table 2.

The other 14 types of PSOs of $\mathbb{E}^5$ are double rotation-reflections or double rotation-inversions. They generate 14 cyclic nonpolar cr PSGs of $\mathbb{E}^5$. For these groups we propose a general symbol: *the symbol of the generator with a double line above.*

Table 2. *WPV symbols of the* 38 *crystallographic cyclic PSGs of* $\mathbb{E}^5$

The first column indicates one possible generator and the second one the WPV symbol of the generated cyclic group.

PSO	WPV symbol of the generated PSG
1	1
2_{xy}	2
3^1_{xy}	3
4^1_{xy}	4
6^1_{xy}	6
$\bar{1}_{xyzt}$	$\bar{1}_4$
$2_{xy}4^1_{zt}$	24
$2_{xy}6^1_{zt}$	26
$3^1_{xy}2_{zt}$	$3\perp 2$
$3^1_{xy}3^1_{\gamma\delta}$	33*
$4^1_{xy}3^1_{zt}$	$4\perp 3$
$4^1_{xy}6^1_{zt}$	46
$4^1_{xy}4^1_{\gamma\delta}$	44*
$6^1_{xy}3^1_{zt}$	63
$6^1_{xy}6^1_{\gamma\delta}$	66*
$5^1_{AB}5^{-2}_{CD}$	55
$10^3_{AB}10^{-1}_{CD}$	1010
$8^1_{AB}8^3_{CD}$	88
$12^1_{AB}12^5_{CD}$	1212
m_u	m
$\bar{1}_{xyz}$	$\bar{1}$
$\overline{3_{xy}}$	$\bar{3}$
$\overline{4_{xy}}$	$\bar{4}$
$\overline{6_{xy}}$	$\bar{6}$
$\bar{1}_5$	$\bar{\bar{1}}$
$3^1_{zt}\bar{\bar{1}}$	$\bar{\bar{3}}$
$4^1_{zt}\bar{\bar{1}}$	$\bar{\bar{4}}$
$6^1_{zt}\bar{\bar{1}}$	$\bar{\bar{6}}$
$3^1_{xy}3^1_{zt}\bar{\bar{1}}$	$\overline{\overline{33}}$
$4^1_{xy}4^1_{zt}\bar{\bar{1}}$	$\overline{\overline{44}}$
$4^1_{xy}3^1_{zt}\bar{\bar{1}}$	$\overline{\overline{43}}$
$6^1_{xy}3^1_{zt}\bar{\bar{1}}$	$\overline{\overline{63}}$
$6^1_{xy}4^1_{zt}\bar{\bar{1}}$	$\overline{\overline{64}}$
$6^1_{xz}6^1_{zt}\bar{\bar{1}}$	$\overline{\overline{66}}$
$5^1_{AB}5^2_{CD}\bar{\bar{1}}$	$\overline{\overline{55}}$
$8^{-3}_{AB}8^{-1}_{CD}\bar{\bar{1}}$	$\overline{\overline{88}}$
$10^{-3}_{AB}10^1_{CD}\bar{\bar{1}}$	$\overline{\overline{1010}}$
$12^{-5}_{AB}12^{-1}_{CD}\bar{\bar{1}}$	$\overline{\overline{1212}}$

These PSGs are listed in the second part of Table 2.

To clarify these notions, we write the elements of three groups:

(*a*) The cr PSO 8^18^3 (88 for short) generates the cyclic cr PSG denoted 88 which has the elements*

$$8^18^3;\ 4^14^{-1};\ 8^38^1;\ \bar{1}_{4(xyzt)};\ 8^58^{-1};\ 4^{-1}4^1;\ 8^{-1}8^{-3};\ 1.$$

* Instead of 8^18^3 we can write 8^18^5; these two elements generate two isomorphic cyclic groups. But if we consider the hypercube of the space $\mathbb{E}^4$ these two PSOs generate two different subgroups of its PSG; they belong to two classes of conjugate elements of the rotation group and to only one for the PSG (Veysseyre, Weigel, Phan & Effantin, 1984).

Obviously we suppose that an orientation of the space has been defined.

(*b*) The cr PSO $\overline{\overline{8^18^3}}$ (*i.e.* $8^18^3\bar{1}_5 = 8^18^3\bar{\bar{1}}$) generates the cyclic cr PSG denoted $\overline{\overline{88}}$ which has for elements

$$\overline{\overline{8^18^3}};\ 4^14^{-1};\ \overline{\overline{8^38^1}};\ \bar{1}_{4(xyzt)};\ \overline{\overline{8^{-3}8^{-1}}};\ 4^{-1}4^1;\ \overline{\overline{8^{-1}8^{-3}}};\ 1.$$

(*c*) Lastly the PSO $\bar{4}^1$ generates the cyclic cr PSG $\bar{4}$ of order 4 which has the elements

$$\bar{4}^1;\quad 2;\quad \overline{4^{-1}};\quad 1.$$

Now, we compare the PSG $\overline{\overline{88}}$ of order 8 and the PSG $88\perp m$ of order 16; the elements of this last one are the eight elements of the PSG 88 and the following ones:

$$8^18^3m_u$$
$$= \overline{\overline{8^{-3}8^{-1}}};\ \overline{\overline{4^{-1}4^1}};\ \overline{\overline{8^{-1}8^{-3}}};\ \bar{1}_5;\ \overline{\overline{8^18^3}};\ \overline{\overline{4^14^{-1}}};\ \overline{\overline{8^38^1}};\ m_u.$$

We remark that the cyclic PSG $\overline{\overline{88}}$ is a subgroup of index 2 of the (non-cyclic) PSG $88\perp m$, as 88 is a subgroup of index 2 of the PSG $88\perp m$. The PSG $88\perp m$ is a hemihedry of the crystal family denoted as *right hyperprism based on 'di-isosquares monoclinic'* (Weigel *et al.*, 1987).

In the same way in the space $\mathbb{E}^3$, the two PSGs 6 and $\bar{6}$ are two subgroups of index 2 of the PSG $6/m$; besides it is possible to write $6\perp m$ for the PSG $6/m$, its 12 elements being 6^1, 3^1, 2, 3^2, 6^5, 1, $\bar{3}^1$, $\bar{6}^1$, $\bar{1}$, $\bar{6}^5$, $\bar{3}^{-2}$, m. Moreover, $6/m$ is a hemihedry of the hexagonal family which has the PSG $6/mmm$ or $6/m\ 2/m\ 2/m$ of order 24 as holohedry.

III. Concluding remarks

Though the number of cr point groups of $\mathbb{E}^5$ is unknown, we describe in this paper all those PSGs which are cyclic; there are 38 of them. We know the polar PSGs of $\mathbb{E}^5$ too: there are 227 polar PSGs in $\mathbb{E}^5$. Their WPV symbols have already been given (Weigel *et al.*, 1987) for they are identical to the 227 PSGs of $\mathbb{E}^4$, Among these, 24 are cyclic (and polar).

In a subsequent paper, we shall give the WPV symbols of the 23 holohedries of the 32 crystal families of $\mathbb{E}^5$ as well as the subgroups of these holohedries in order to enumerate all the cr PSGs of $\mathbb{E}^5$. A physical application of these results is the study of the di-incommensurate phases of the physical space.

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